Quantum Tunneling in Oscillatory Driven Double-Well Potential *1

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振動外場下での二重井戸型ポテンシャル中のトンネル効果を考える。外場の振動数成分が増えた場合のトンネル効果を調べた結果、ある成分数迄は規則的遷移が見られ、それを越えると遷移確率はカオス的な振動に転移する傾向を示した。さらに最適制御理論に基づき、その外場も存在する下でのトンネル確率の制御可能性を単色系、多色系の場合について議論する。

1 Introduction and Models

We numerically investigate quantum tunneling in one dimensional double-well system driven by oscillatory external field. It is found that tunneling rate between the wells behaves regularly up to some extent of number of frequency components, while it behaves irregularly above such a extent as the perturbation strength is increased. Moreover we report controllability of quantum dynamics via tunneling in the system based on optimal control theory(OCT).

The model Hamiltonian has the form, $H(p,q,t) = \frac{p^2}{2} + V(q,t)$. And the time dependent potential is $V(q,t) = \frac{q^4}{4} - \left(a - \frac{\epsilon}{\sqrt{M}} \Sigma_{i=1}^{M} \sin \Omega_i t\right) \frac{q^2}{2}$, where a is set to 5. ϵ and M are a perturbation strength and the number of frequency components respectively. $\{\Omega_i\}$ are mutually incommensurate frequencies. Therefore the potential has polychromatic time dependence.

To investigate tunneling dynamics, we take a Gaussian wave packet localized on the right bottom of the double-well potential as the initial state $\psi(q, t=0)$, which approximates the liner combination of the ground state doublet with equal weight. Then we calculate the tunneling rate $P_L(t) = \int_{-\infty}^{0} |\psi(q,t)|^2 dq$ by solving time dependent Scarödanger equation numerically.

2 Tunneling Dynamics

We show numerical results on tunneling dynamics in the above system [1, 2]. Fig. 1 shows tunneling dynamics for some cases. Following this figure it is found that tunneling dynamics in a polychromatic case behaves regularly for a small perturbation strength but it becomes chaotic for a relatively large value of the perturbation strength. Therefore it is expected that there exists *transition region* on the parameter space (ϵ, M) where destruction of coherent tunneling occurs.

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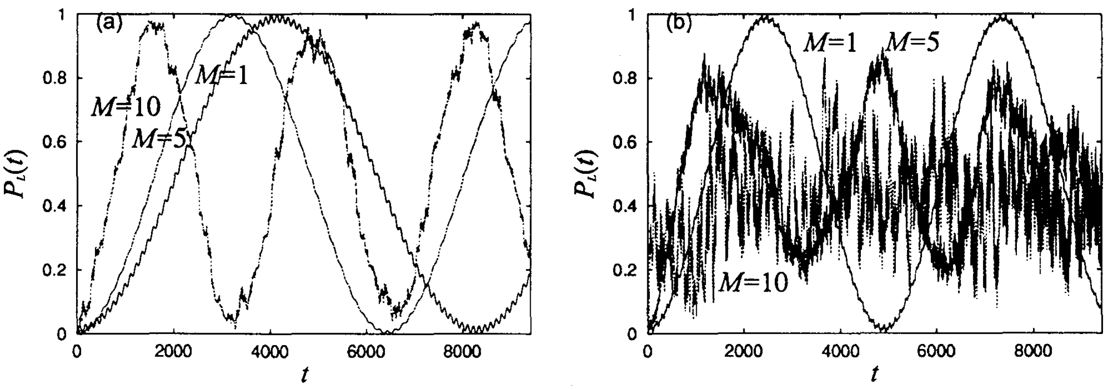

Fig. 1: The panel (a) shows in case of $\epsilon = 0.2$, and panel (b) in case of $\epsilon = 0.5$, for $M = 1, 5, 10$.

3 Control of Quantum Dynamics via Tunneling

We consider controllability of wave packet tunneling in the above systems based on optimal control theory [3]. Let $E(t)$ be another external field to control the dynamics, and the total Hamiltonian be $H_{tot}(p, q, t) = H(p, q, t) + qE(t)$. Then in order to maximize the overlap $\psi(x, t)$ at the target time t_f and the target state which is localized on the left well(reflection of the initial state), the external field is determined by OCT. (We need iteration method to get numerical solution of the external field.) Fig. 2 shows the numerical results on control of tunneling in short time regime $t \simeq 300 (= t_f)$ in case of $\epsilon = 0.5$. Although tunneling rate is nearly zero around t_f in all cases, it is found that almost perfect optimization can be achieved at a few iteration step in all cases.

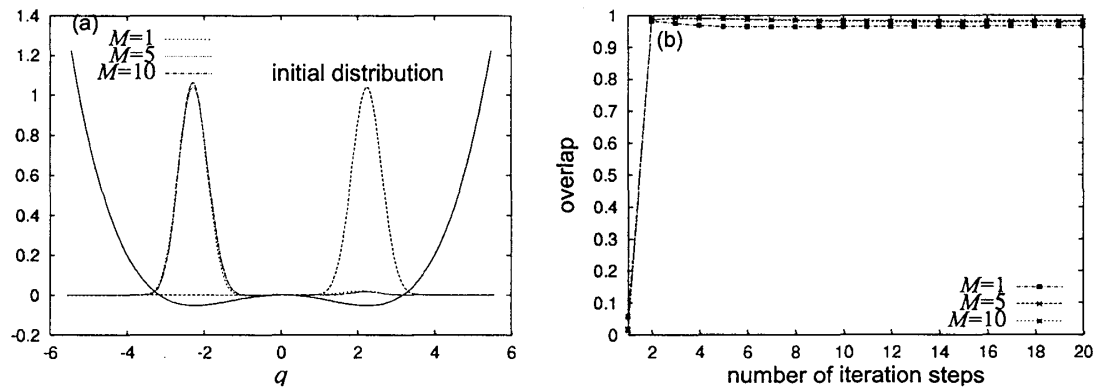

Fig. 2: The panel (a) shows some distributions at the target time t_f. The outer curve represents the potential at $t = 0$. The panel (b) shows overlap vs number of iteration steps.

Referrences

[1] Akira Igarashi and Hiroaki Yamada, Chem. Phys. *in press.*
[2] Akira Igarashi and Hiroaki Yamada, in preparation;
[3] Akira Igarashi and Hiroaki Yamada, Phys. Stat. Sol. (c) **1** (2004), 2812 [*3]

[*3] The figures of stochastic case in the paper have some mistakes.